VELOCITY PROFILE EQUATIONS FOR MICROVESSEL BLOOD FLOW IN MAMMALS / EQUAÇÕES DO PERFIL DE VELOCIDADE APLICADAS AO FLUXO MICROVASCULAR EM MAMÍFEROS

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ABSTRACT

A mini review of the available velocity profile equations for microvessel blood flow is presented. These equations are divided into two groups and then a preliminary assessment of the first group is demonstrated. Finally, there is a discussion on how these equations could be applied to human microcirculation *in vivo*.

INTRODUCTION

Before the last third of the 20th century, the only equation available to the researchers studying laminar flows inside cylindrical tubes was the parabolic one. In the seventies it became evident that the flow of blood is quite different from simple Newtonian flows like that of water.

Blood exhibits special shear thinning properties due to reasons which are still partly unexplained. For example the molecular and biochemical basis of the rouleaux (structures resembling coin piles) formation is still unknown. The shear thinning property means that blood viscosity diminishes (blood becomes thinner) as shear rate increases. For the case of blood, this property is quite evident taking into account that at high shear rates (> 100 s^{-1}) its viscosity is many times lower than that at shear rates below 10 s^{-1}. This means that near the vessel axis where there are such low shear rates, blood is much more viscous, causing a characteristic "blunting" of the velocity profile[2,7,8,12,13,15].

After 1970, the researchers in the field proposed equations trying to describe the blunting of the velocity profile in the microvasculature with diameters (D) higher than 20 μm. The diametric down size limit of the 20 μm[1] is imposed by the manifestation of the biphasic nature of blood in the smallest arterioles and venules and the capillaries. In these microvessels red blood cells (RBCs) flow separately, constituting a different liquid

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phase from plasma and therefore the flow medium can not be considered as a "continuum" and a velocity profile can not be defined in the ordinary sense.

The available velocity profile equations today, could be divided in many ways, but in this review the criterion was whether they can be easily reduced to the classic parabolic equation (Group A) or not (Group B).

Available equations

Realistic assumptions of the blood flow in straight sections of microvessels with D > 20μm, several diameters downstream their blood flow entrance, are: 1) incompressible flow, 2) continuous medium, 3) viscous flow with Reynolds number less than 1, 4) non-Newtonian medium, 5) cylindrical vessel geometry and 6) axisymmetric velocity profile with maximum velocity V_m on the vessel axis.

All the above conditions are satisfied by the equations presented below except for the parabolic equation which can not satisfy the non-Newtonian condition.

Group A

This group comprises 3 velocity profile equations:

$$V(r) = V_m \left[1 - \left(\frac{r}{R}\right)^2\right] \quad (1)$$

$$V(r) = V_m \left[1 - \left(\frac{r}{R}\right)^{\kappa}\right] \quad (2)$$

$$V(r) = V_m \left[1 - \kappa_1 \left(\frac{r}{R}\right)^2\right]\left[1 - \left(\frac{r}{R}\right)^{\kappa_2}\right] \quad (3)$$

Where V(r) is the velocity at radial position r, for the parabolic (equation 1), the Roevros[10] (equation 2) and the Koutsiaris[4] (equation 3) cases. R is the radius of the cylindrical vessel and V_m is the maximum velocity of the symmetrical profile on the vessel axis.

The parameters κ and κ_1, κ_2 affect the velocity profile shape of equation 2 and 3 respectively. For a velocity profile blunter than the parabolic one and with the same V_m, the following conditions must be satisfied: $\kappa > 2$ (equation 2), $0 < \kappa_1 < 1$, $\kappa_2 > 2$ and $(1-\kappa_1)\,\kappa_2 > 2$ (equation 3).

In equation 2, the higher the κ, the flatter the profile near the vessel axis and the higher the wall shear rate[10]. In equation 3, the advantage is that the bluntness of the profile can be controlled independently near the axis and the wall respectively: as κ_1 approaches zero the profile becomes flatter near the axis and as κ_2 takes values higher than 2 the profile becomes flatter near the wall[4].

As it was mentioned in the introduction, equations 2 and 3 can be reduced to the parabolic equation when: $\kappa = 2$ (equation 2) and $\kappa_1 = 0$, $\kappa_2 = 2$ (equation 3).

It should also be noted that equation 1 can be derived from basic physical principles whereas equations 2 and 3 are empirical.

Group B

Damiano *et al* [2] made significant contributions in theory and experi-

mental measurements and provided a way to estimate the viscosity profile as well as the effective viscosity *in vivo*. They proposed the following velocity profile equation which identically satisfies the momentum equation and boundary coonditions[7]:

$$V(r) = V_m \frac{\int_{r/R}^{R} f(\sigma)\, d\sigma}{\int_{0}^{R} f(\sigma)\, d\sigma} \qquad (4)$$

Where $f(\sigma)$ is a function of R and of two independent parameters c_1 and c_2 which can be found through non linear regression analysis that uses equation 4 to minimize the least-squares error (SSE) of the fit to the experimental velocity profile data sets[7]. With a suitable modification the equation can take into account the infinitesimal flow inside the microvascular glycocalyx layer. This is important for the transcapillary exchange and endothelium studies but from the volume flow estimation point of view it contributes little due to the very low velocities near the vessel walls.

The profile of this equation will not be shown here because the parameters c_1 and c_2, were not given. However, the fine fitting of equation 4 to experimental data from mouse venules can be seen in the papers of Damiano *et al* [2] and Long *et al* [7].

Available data in the literature

The actual measurement of the blood velocity profile in microvessels is a known difficult task to accomplish. This is in part due to the many different scientific fields that need to cooperate and in part due to the expensive experimental set up needed.

Therefore it is not surprising that apart from some preliminary efforts in the 70's[12], the first *in vivo* quality velocity profile measurements were presented in 1986 by the group of professor Reneman[15]. They measured velocity profiles in the arterioles of the rabbit mesentery using as flow tracers platelets labeled with a special fluorescence technique.

More than 15 years later, Nakano *et al* [8] and Sugii *et al* [13] measured the velocity profile in arterioles of the rat mesentery using a technique introduced 2 years earlier by Sugii *et al* [14] under the name "high resolution particle image velocimetry" (HR-PIV). This technique was an improved combination of previous micro PIV volume illumination techniques[3,11] and provided an excellent spatial resolution of 0.8 μm.

In 2004, Long *et al* [7] (12 venules from male mice, 24 μm ≤ D ≤ 42.9 μm) and Damiano *et al* [2] (9 light-dye treated venules from 3 mice, 24 μm ≤ D ≤ 42.9 μm) provided the most complete velocity profile measurements until now, with the best spatial resolution. They measured in the cremaster muscle of mice using fluorescent microspheres (0.47 ± 0.01 μm) as blood flow tracers. More recently, Potter and Damiano[9] performed measurements in mice venules up to diameters of 101 μm but they mainly concentrated on the properties of the endothelium glycocalyx layer both *in vivo* and *in vitro*.

Efficiency

A recent preliminary evaluation[5] of the group A equations, assuming

they all have the same axial velocity and using 8 experimental profile data from the literature[7], showed that the parabolic equation tends to underestimate blood velocity, reaching a maximum relative error of – 72% near the vessel wall; therefore, the parabolic equation leads to an average volume flow underestimation of approximately – 18%. The Roevros equation tends to overestimate blood velocity reaching a maximum relative error of + 48% at a radial position between 70% and 80% of the vessel radius R; therefore, the Roevros equation leads to an average volume flow overestimation of approximately + 20%. The Koutsiaris equation tends to approximate blood velocity with a relative error between – 8% and + 7%, for all radial positions, leading to an average volume flow error of less than 0.5%.

In order to compare the 3 equations, fixed values for the parameters were selected[4,5]: $\kappa = 9$ (equation 2) and $\kappa_1 = 0.58$, $\kappa_2 = 22$ (equation 3). An example of the velocity distributions given by the 3 equations is shown in Fig. 1.

In conclusion, equations 1 and 2 lead to a severe underestimation and overestimation of the blood velocity respectively, but equations 3 and 4 approximate quite well the velocity profile of mouse blood (examples of equation 4 can be seen elsewhere[2,7]). However, equation 3 requires only measurement of the axial velocity.

Application to humans

A fundamental remaining question is whether equations 3 and 4, which seem to approximate efficiently the blood velocity profile in the mouse microcirculation, can be applied in the human microcirculation also. Taking into account the current

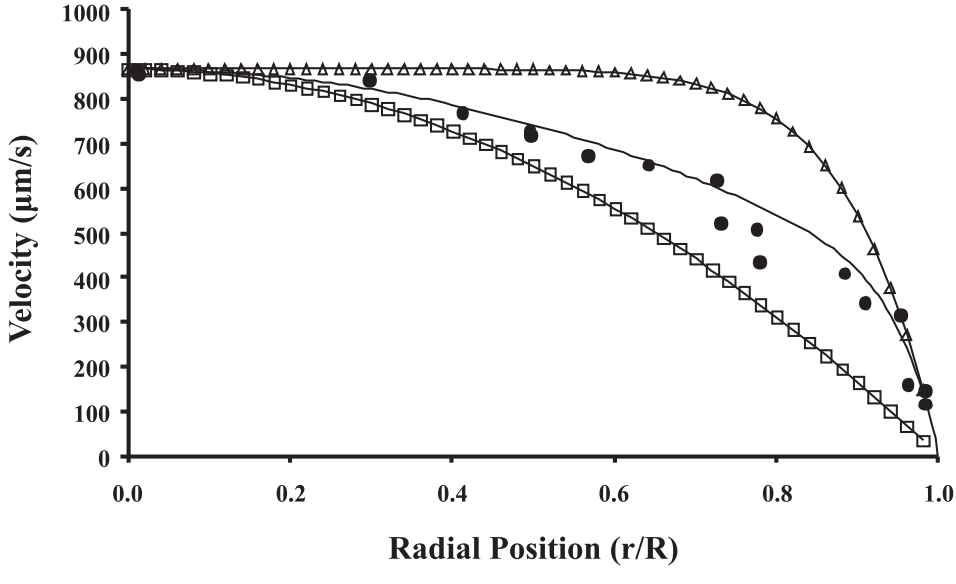

Fig. 1* – The velocity profile equations 1, 2 and 3 (with the same axial velocity Vm) are shown in squares, triangles and solid black line, respectively. The experimental velocity profile data from a 38.6 μm mouse venule (Long *et al*[7]), with their radial position normalized, are shown in black dots. Graph was taken from Koutsiaris *et al*[5].

* For experimental details of Fig. 1, the reader should get access to reference 7 and for equation parameters to references 4 and 5

state of technology, it would be rather difficult now, or in the near future, to measure the velocity profile of blood in human microvessels. An indirect way of finding the answer would be to measure the rheological differences between mouse and human blood and more specifically the viscosity differences at low shear rates. In case these differences prove to be high enough, presumably the profiles will be different and a change of the equation parameters will be required.

It is already known[16] that in humans and other athletic species like horses, whole blood viscosity (WBV) is higher than in mice. However, this WBV difference becomes important at very low shear rates (< 10 s^{-1}) occurring close to the vessel axis, as already was mentioned in the introduction. Using equation 3 shown in figure 1, the shear rate of 10 s^{-1} corresponds to a radial position r = 0.18 R, or to a surface area of only 3.3% of the total cross sectional area of the vessel. Therefore, given that the important WBV difference concerns only a small portion of the vessel cross sectional area, it is logical to assume that equations 3 and 4 could be applied to the human microvessels as well.

A more detailed viscometric experiment comparing mouse and human blood would involve WBV measurements at many different shear rates with emphasis on shear rates below 10 s^{-1} at physiologic temperatures (human blood at 36.6 ^{o}C and mouse blood at 38 ^{o}C).

One probable complication would be the selection of the appropriate hematocrit level since according to the Fahraeus effect, the average microcirculatory hematocrit is lower than the systemic hematocrit (Hs). So, for diameters of approximately 20 μm, the average hematocrit would be 0.28Hs in the venous side and higher in the arteriolar side[6]. A suggested set of experiments would comprise WBV measurements at hematocrits of .28Hs, .38Hs and .50Hs.

CONCLUSION

A mini review of the current velocity profile equations for the description of blood flow in microvessels was presented. There are now 2 velocity profile equations which approximate quite well the actual microvessel velocity profile of mice and logically they could also be used in humans. What seems to be more than certain now is that the classic parabolic velocity profile is inappropriate for use in mammals.

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